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Spin waves in stripes submitted to a perpendicular applied field

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Abstract
In the dipolar approximation, we present an analytical study of the magnetic modes occurring in a stripe submitted to a magnetic field **H** parallel to a principal axis of its elliptical cross section. The studied excitations oscillate within the cross section. The results differ greatly from those obtained in the previously treated case of **H** parallel to the stripe direction. The dependences of the frequencies upon the applied field are opposite below and above the saturation field H_s, where H_s defines the limit between the oblique phase ($H < H_s$) and the aligned one ($H > H_s$): the frequencies slow down until H_s, they vanish at H_s and they increase above. This softening was previously observed on stripes with a rectangular cross section and the measured critical field lies near the saturation field, in agreement with our calculations.

1. Introduction

Many experimental and theoretical studies have been performed during recent years on spin waves in magnetic stripes [1–10]. When the field is applied along the stripe, spin waves oscillating within the cross section were observed and approximately described with the help of an effective quantized wavevector $Q \approx n\pi/w$, where w is the width of the stripe section, for w sufficiently larger than its thickness t; n stands for any positive integer. In addition, for this geometrical arrangement, a complete analytical approach is available in the dipolar approximation, but only for an elliptical section [11]. When an in-plane field is applied perpendicularly to the stripe, spin waves are also observed; they are described through numerical approaches [3–9]. Some of the evidenced modes can be regarded as quantized backward propagative excitations; others are appropriately described in terms of modes localized near the edges of the rectangular section of the studied samples, a consequence of the lack of homogeneity of the internal field. In the present paper we propose, in this perpendicular geometry, a complete analytical approach for stripes with an elliptical cross section, in the absence of anisotropy and of exchange. Indeed, the modes obtained using this dipolar approximation are not localized, since the internal static magnetic field is uniform.

In some sense, we show that they can be regarded as quantized backward modes. Our analytical results enable us to present a physical discussion concerning the variations of the eigenfrequencies as functions of the applied field.

2. Equations and notations

We study stripes with circular or elliptical cross section. At least concerning the static equilibrium magnetization **M**, they can be considered as elongated ellipsoids parallel to the z-direction, submitted to a magnetic field **H** applied in the x-direction of one of the principal axes (length: $2a$) of the elliptical cross section; the other axis (length: $2b$) lies along the y-direction. The circular case corresponds to $a = b = R$. The equilibrium equation can be written

$$\mathbf{M} \times \mathbf{H}_\mathrm{i} = \mathbf{0} \tag{1}$$

where $\mathbf{H}_\mathrm{i}$ stands for the internal field. The unique component of $\mathbf{H}_\mathrm{i}$ is written as

$$H_\mathrm{i} = H - 4\pi N_x M \qquad \text{with } N_x = b/(a+b). \tag{2}$$

(Notice that, in the case of any ellipsoid, one defines three demagnetizing factors N_x, N_y and N_z with $N_x + N_y + N_z = 1$; here, $N_z = 0$, $N_x = b/(a+b)$, $N_y = a/(a+b)$). Depending upon the value of the applied field H, the stripe is *unsaturated* or *saturated*:

$H < H_\mathrm{s} = 4\pi N_x M \Rightarrow$ unsaturated stripe:

$$M_x = H/(4\pi N_x), \qquad M_z = (M^2 - M_x^2)^{1/2}, \qquad H_\mathrm{i} = 0 \tag{3a}$$

$H > H_\mathrm{s} = 4\pi N_x M \Rightarrow$ saturated stripe:

$$M_x = M, \qquad M_z = 0, \qquad H_\mathrm{i} = H - 4\pi N_x M. \tag{3b}$$

The dynamic magnetization **m** obeys the Landau Lifshitz equation. In the dipolar approximation,

$$\mathrm{i}\Omega\mathbf{m} = \mathbf{M} \times \mathbf{h}_\mathrm{i} + \mathbf{m} \times \mathbf{H}_\mathrm{i} \tag{4}$$

where $\Omega = \omega/\gamma$ is the ratio of the pulsation to the gyromagnetic factor and where $\mathbf{h}_\mathrm{i}$ is the dynamic demagnetizing field. In the quasi-static approximation, $\mathbf{h}_\mathrm{i} = \nabla\phi$, where the potential ϕ is related to the dynamic magnetization **m** by

$$\nabla \cdot (\nabla\phi + 4\pi m) = 0. \tag{5}$$

Outside of the magnetic object, the dynamic field is $h_\mathrm{e} = \nabla\psi$, where

$$\Delta\psi = 0. \tag{6}$$

Thus the boundary conditions are

$$\psi = \phi \tag{7a}$$

and

$$\mathbf{n} \cdot \nabla\psi = \mathbf{n} \cdot (\nabla\phi + 4\pi\mathbf{m}) \tag{7b}$$

where **n** is a vector normal to the boundary.

In the following we are interested with magnetic excitations non-propagating along the z-axis, and, consequently, with oscillating excitations independent of z.

3. Unsaturated stripes

We first study the eigenmodes of a circular cylinder, in order to introduce the physical features of these excitations on this simple example. Then, following the same method, we derive the eigenfrequencies in an elliptical stripe. Finally, we show that the uniform mode, which is easily found directly, belongs to the set of our obtained solutions.

For unsaturated stripes, equation (4) simply reduces to

$$\mathrm{i}\Omega\mathbf{m} = \mathbf{M} \times \mathbf{h}_{\mathrm{i}} \tag{8}$$

since $H_{\mathrm{i}} = 0$.

It results from equations (5) and (8) that

$$\Delta\phi = 0. \tag{9}$$

3.1. Circular cylinder

Suitable solutions for ϕ and ψ can be written as

$$\phi = f_0(r/R)^n \exp(\mathrm{i}n\theta) \tag{10a}$$

$$\psi = g_0(R/r)^n \exp(\mathrm{i}n\theta) \tag{10b}$$

where n is any positive integer and where f_0 and g_0 are appropriate constants; the above choice prevents the divergence of the potential. Using equation (8), it immediately results from equations (7) that

$$\Omega = 2\pi M_z = ((2\pi M)^2 - H^2)^{1/2}. \tag{11}$$

The frequency does not depend on the n value. It decreases from $2\pi M$ to 0 when H increases from 0 to $H_{\mathrm{s}} = 2\pi M$.

3.2. Elliptical section

We search for solutions with an angular dependence proportional to $\exp(\mathrm{i}n\theta)$ on the stripe boundary $(a\cos\theta, b\sin\theta)$. Inside of the stripe the functions of the form

$$\phi = f_1(x + \mathrm{i}y + ((x + \mathrm{i}y)^2 - a^2 + b^2)^{1/2})^n + f_2(x - \mathrm{i}y + ((x - \mathrm{i}y)^2 - a^2 + b^2)^{1/2})^n \tag{12}$$

adequately fit the above condition. Here, and in the following, the defined square root is positive when it is real or shows a positive imaginary part when it is complex. Outside of the stripe, to prevent the divergence of the potential at infinity, one has to choose:

$$\text{for } y < 0 \qquad \psi = g(x + \mathrm{i}y + ((x + \mathrm{i}y)^2 - a^2 + b^2)^{1/2})^n \tag{13a}$$

$$\text{for } y > 0 \qquad \psi = g(x - \mathrm{i}y + ((x - \mathrm{i}y)^2 - a^2 + b^2)^{1/2})^n. \tag{13b}$$

The boundary conditions provide relations between the three constants f_1, f_2 and g. Finally, it results that

$$\Omega = 2\pi M_z(1 - ((a - b)/(a + b))^{2n})^{1/2}$$
$$\text{with: } 2\pi M_z = ((2\pi M)^2 - (H(a + b)/2b)^2)^{1/2}. \tag{14}$$

Now the frequency depends on n. It decreases from $2\pi M(1 - ((a - b)/(a + b))^{2n})^{1/2}$ to 0 when H increases from 0 to H_{s}.

It is important to point out that the chosen form for ϕ in equation (12) leads to an non-physical divergency of the oscillating magnetization at the points $\{x = \pm(a^2 - b^2)^{1/2}, y = 0\}$, i.e. at the focuses of the elliptic cross section: the components of $\mathbf{m}$ being related to the derivative of ϕ versus x or versus y, they automatically diverge at these points. As a result, it is not possible to derive the profiles of the studied modes with the help of the obtained expressions for $\partial\phi/\partial x$ and $\partial\phi/\partial y$. Anyway, the above formalism correctly accounts for the frequencies [10], as often verified using, for instance, numerical calculations based on the finite elements method [1, 6, 9]. Indeed, the numerical methods also allow us to estimate the profiles [9].

3.3. The uniform mode

For an ellipsoid, one of the exact solutions of the spin waves problem consists in an uniform mode. It is immediately found from the equation of motion (8) in the unsaturated case, noticing that $h_{ix} = -4\pi N_x m_x$, $h_{iy} = -4\pi N_y m_y$, $h_{iz} = -4\pi N_z m_z$. The obtained frequency

$$\Omega = 4\pi M_z((N_y - N_z)(N_x - N_z))^{1/2} \qquad \text{with } M_z = (M^2 - (H/(4\pi(N_x - N_z)))^2)^{1/2} \tag{15}$$

provides for the studied case of a cylinder ($N_z = 0$):

$$\Omega = 4\pi M_z(N_y N_x)^{1/2} \qquad \text{with } M_z = (M^2 - (H/4\pi N_x)^2)^{1/2} \tag{16}$$

which can be written

$$\Omega = ((2\pi M)^2 - (H(a+b)/2b)^2)^{1/2}(4ab/(a+b)^2)^{1/2}. \tag{17}$$

This is the solution provided by equation (11) in a circular cylinder and by equation (14) in an elliptical cylinder for the mode $n = 1$. In addition, we have now established that it is uniform; consequently, it should be easily observable on applying a uniform rf field.

4. Saturated stripes

4.1. General study

The case of a circular cylinder does not provide any significant simplification. Indeed, it is simply an instance of the general problem of an elliptic section assuming $a = b = R$.

The presence of a static internal field prevents $\Delta\phi$ from vanishing. Defining λ by

$$\lambda^2 = 4\pi M H_i/(\Omega^2 - H_i^2) - 1 \tag{18}$$

it results from equations (4) and (5) that

$$\partial^2\phi/\partial x^2 - \lambda^2\partial^2\phi/\partial y^2 = 0. \tag{19}$$

The general solution of equation (9) is a linear combination of two functions of $(\lambda x + y)$ and $(\lambda x - y)$, respectively. Again, we search for potentials ϕ and ψ with an angular dependence proportional to $\exp(\mathrm{i}n\theta)$ on the stripe boundary. Inside the stripe,

$$\begin{aligned}\phi = {} & f_1(\lambda x + y + \mathrm{i}(-(\lambda x + y)^2 + \lambda^2 a^2 + b^2)^{1/2})^n \\ & + f_2(\lambda x - y + \mathrm{i}(-(\lambda x - y)^2 + \lambda^2 a^2 + b^2)^{1/2})^n\end{aligned} \tag{20}$$

while, outside the stripe, ψ remains given by equations (13). The boundary conditions give

$$((\lambda - i)/(\lambda + i))^2 = ((\lambda a + \mathrm{i}b)/(\lambda a - \mathrm{i}b))^{2n}. \tag{21}$$

Introducing α by

$$\tan[\alpha] = (b/a\lambda) \tag{22}$$

equation (21) is equivalent to

$$\tan[\alpha] = -(b/a)\tan[n\alpha] \tag{23a}$$

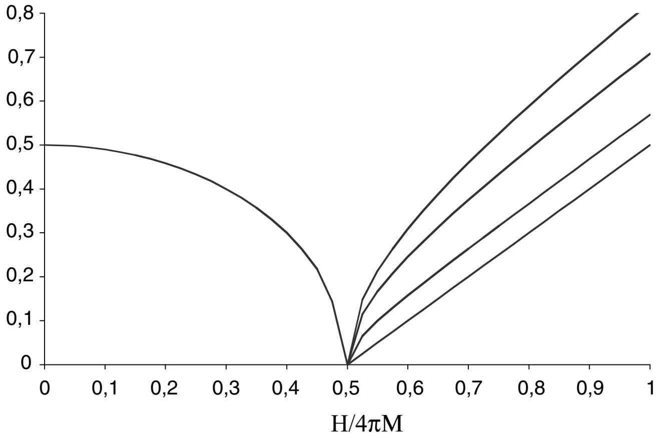

Figure 1. Field dependence of the frequencies of spin waves in a stripe with a circular cross section. Above H_s, for each n value there are $(n+1)$ distinct branches (see the text). Here, $n = 3$.

or

$$\tan[\alpha] = (b/a)\cot[n\alpha]. \tag{23b}$$

The solutions of equations (23*a*) and (23*b*) provide the frequencies of the magnetic modes:

$$\Omega = (H_i^2 + 4\pi M H_i/(1+\lambda^2))^{1/2} = (H_i^2 + 4\pi M H_i a^2/(a^2 + b^2(\cot[\alpha])^2))^{1/2} \tag{24}$$

with:

$$H_i = H - 4\pi(b/(a+b))M. \tag{25}$$

The set of equations (23) generally shows $(n+1)$ solutions (including $\alpha = 0$, which gives rise to the frequency H_i).

Here again let us notice that the chosen form of ϕ in equation (20) prevents its use for the evaluation of the profiles, as a consequence of non-physical divergencies occurring on the four lines

$$y = \pm(b/\tan[\alpha])(x/a \pm (1 + (\tan[\alpha])^2)^{0.5}) \tag{26}$$

which are tangential to the ellipse defining the cross section.

For every mode the frequency vanishes at $H = H_s$ and increases versus H. The solutions of equation (23*a*) correspond to $f_2 = f_1$ (even solutions), while the solutions of equation (23*b*) provide $f_2 = -f_1$ (odd solutions). The increasing $(n+1)$ values of α alternate solutions of equation (23*a*) and of equation (23*b*). The symmetry properties of each mode depend upon the involved equations ((23*a*) or (23*b*)) and upon the parity of n. The behaviour of the components of the oscillating magnetization when changing x into $-x$ and/or y into $-y$ are listed in table 1. Examples of frequency variations versus H are shown in figures 1 and 2 for various geometries (circular cylinder and elliptical cylinder with $b/a = 1/9$). The symmetry properties of the modes are correctly described but, as noticed above, the profiles have to be found using a numerical method. In figure 3, we show some profiles for $|m_x|^2$ and $|m_y|^2$ that we obtain using a previously described technique of finite elements [9].

Finally, in view of a qualitative discussion concerning the studied excitations, it is interesting to regard them as approximately described by propagative waves showing a quantized wavevector $Q_\parallel$. For other different geometrical arrangements, the validity of such an approach was previously experimentally and numerically [1–4] justified in the case of

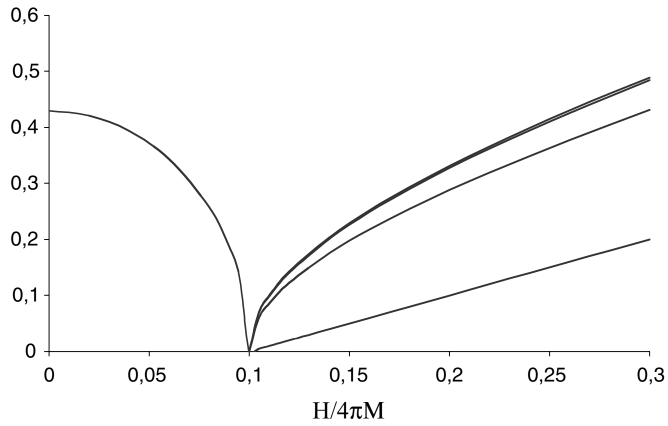

Figure 2. Field dependence of the frequencies of spin waves in a stripe with an elliptic cross section ($b/a = 1/9$). Above H_s, for each n value, there are $(n+1)$ distinct branches (see the text). Here, $n = 3$. With the chosen value of b/a, the two highest branches are practically identical.

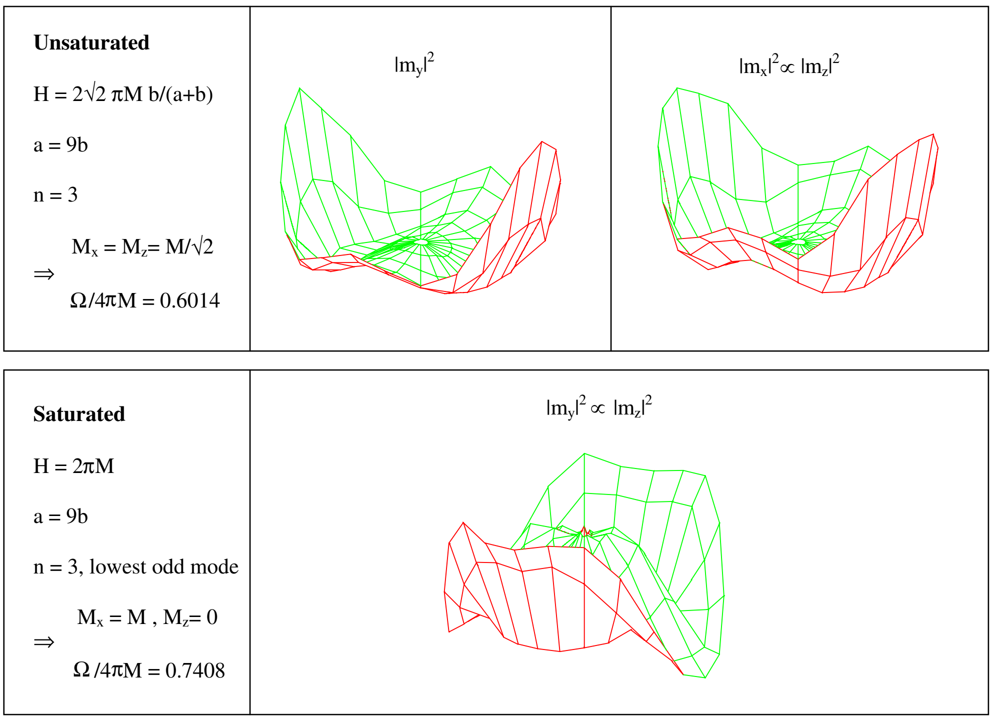

Figure 3. Dynamic magnetization profiles calculated using a finite elements method [9]. The b/a ratio used for calculations ($b/a = 1/9$) is amplified in the pictures. *Top line*: *unsaturated case*; $|m_y|^2$ (left) and $|m_z|^2$ (right) variations are shown for $n = 3$ with $H/4\pi M = b/((a+b)\sqrt{2})$; it is easy to prove that m_x/m_z is constant across the section. *Bottom line*: *saturated case*; the $|m_z|^2$ variation is shown for $n = 3$ with $H/4\pi M = 0.5$; it is easy to prove that m_x vanishes and that m_y/m_z is constant across the section.

(This figure is in colour only in the electronic version)

Table 1. Symmetries of the components of the oscillating magnetization for the spin waves in a saturated stripe.

	$x \to -x$	$y \to -y$
$n = 2p$, even modes	$m_y \to m_y, m_z \to -m_z$	$m_y \to -m_y, m_z \to -m_z$
$n = 2p$, odd modes	$m_y \to -m_y, m_z \to m_z$	$m_y \to m_y, m_z \to m_z$
$n = 2p + 1$, even modes	$m_y \to -m_y, m_z \to m_z$	$m_y \to -m_y, m_z \to -m_z$
$n = 2p + 1$, odd modes	$m_y \to m_y, m_z \to -m_z$	$m_y \to m_y, m_z \to m_z$

rectangular cross section and was theoretically demonstrated in the case of elliptical cross section: in the first case the allowed values of $Q_\parallel$ do not markedly differ from $n\pi/w$; a naive extrapolation leads to quantized wavevectors equal to $2n\pi/a$ for an elliptical section, but rigorous calculation provides a significantly different result, namely n/a. In the cases mentioned above, $\mathbf{Q}_\parallel$ was perpendicular to the internal field and the frequency dispersion of the comparative continuous film was appropriate to this geometry. In the present geometrical arrangement where the magnetic oscillation takes place in the plane of the cross section, the required comparative dispersion corresponds to a wavevector $\mathbf{Q}_\parallel$ parallel to the internal field $\mathbf{H}_\mathrm{i}$ and is associated to backward propagative spin waves in a continuous film. Its expression can be written [12]

$$\Omega = (H_\mathrm{i}^2 + 4\pi M H_\mathrm{i}/(1 + (\tan[tQ_\perp/2])^2))^{1/2} \tag{27}$$

where the quantity $Q_\perp$ is the solution of the equation

$$\tan[tQ_\perp/2] = Q_\parallel/Q_\perp. \tag{28}$$

Equations (24) and (27) become identical if

$$(b/a)\cot[\alpha] = \tan[tQ_\perp/2]. \tag{29}$$

Identifying $t/2$ with b and assuming that α and $(bQ_\perp)$ have small values, it results from equations (28) and (29) that

$$Q_\parallel \approx bQ_\perp^2 \approx b/(a^2\alpha^2). \tag{30}$$

The solutions corresponding to equation (23b) ensure, for small values of α,

$$\alpha \approx (b/na)^{1/2}. \tag{31}$$

Hence, finally one finds

$$Q_\parallel \approx n/a = 2n/w. \tag{32}$$

The effective quantized wavevector has the same expression as in the previously studied geometry. However, this correspondence is less useful for the presently discussed parallel geometry than for the perpendicular one. In this last case the continuous film shows only one dispersion curve: any mode in the film is completely defined when $Q_\parallel$ is specified and any mode in the stripe is completely defined when n is specified; the appropriate quantization of $Q_\parallel$ provides a complete description of the stripe modes. In the parallel geometry such a bi-univocal relation does not subsist: first, in the stripe any n value gives rise to $(n+1)$ distinct modes and, second, in the continuous film it is evident from relation (28) that any $Q_\parallel$ value gives rise to an infinite set of solutions for $Q_\perp$ and, consequently, to an infinite set of modes. Strictly speaking, the one to one correspondence is lost.

4.2. The uniform mode

For the saturated case, the uniform mode is easily found, in the same way as for the unsaturated case: for a cylinder ($N_z = 0$) one immediately finds

$$\Omega = ((H + 4\pi(N_y - N_x)M)(H - 4\pi N_x M))^{1/2}. \tag{33}$$

It results that

$$\Omega = ((H + 4\pi M(a - b)/(a + b))(H - 4\pi Mb/(a + b)))^{1/2}. \tag{34}$$

We find the frequency given by equation (24) for an odd mode (equation (23*b*)) in the case $n = 1$. The uniform mode belongs to the whole calculated set of solutions, as expected. Here again, in principle, such a profile is suspected to allow an easy observation on applying a rf uniform magnetic field.

5. Conclusion

The exact calculations performed in the simplified case of a stripe showing an elliptical cross section in the dipolar approximation provide a qualitative tool to discuss the available experimental data related to the nearby situations encountered with rectangular cross sections. For the studied parallel geometry, the most striking effect is the softening of the modes in the vicinity of the saturation field. Above this saturation field, the increase of the number of distinct modes is put in evidence. The pseudo-backward character also appears. All these effects have been experimentally observed [7, 8]. For our part, we have recently obtained preliminary results on the softening, using Brillouin scattering in permalloy stripes; a complete experimental study will be published in the future.

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